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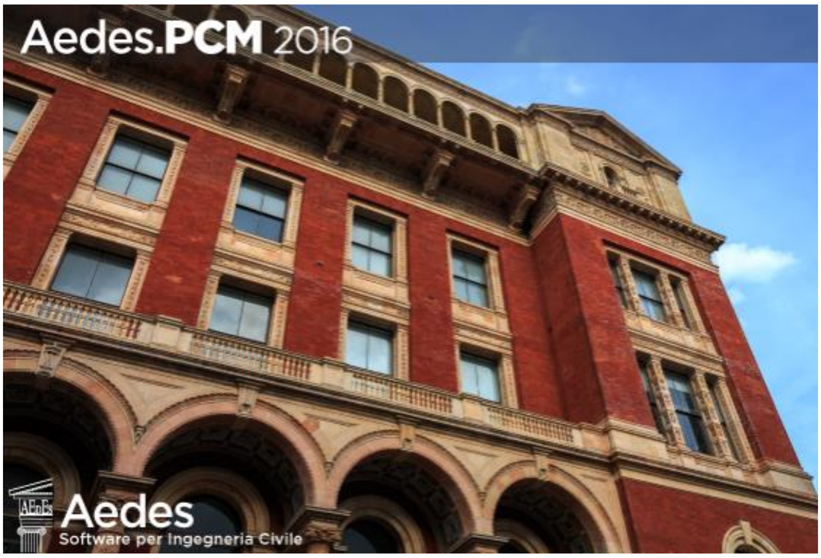

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PROGETTAZIONE DI COSTRUZIONI IN MURATURA

Structural analysis of masonry buildings according to the current Standards

Theory

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1 ANALYSIS OF MASONRY BUILDINGS: FROM POR METHOD TO PUSHOVER

1.1 ABOUT STRUCTURAL ANALYSIS OF MASONRY BUILDINGS

Masonry is a heterogeneous material and its mechanical properties are very different from the ones of its components (masonry elements and mortar) taken separately. Generally, masonry mechanical properties (compressive and shear strength, elastic moduli) can be identified through in-situ or laboratory testing. In case of new masonry, the accurate knowledge of the components characteristics allows to estimate the mechanical properties of the material as a whole with the aid of formulas and tables provided by the Standards [9, 10]. For existing masonry, instead, the issue is much more complex given the broad variety of construction techniques and the impossibility to reproduce the exact configuration of the material for laboratory testing. However, several experimental campaigns were carried out in order to characterize the most common masonry typologies and their results yielded reference values of the mechanical parameters, now provided by the Standards [10].

Masonry buildings are box structures consisting of walls stressed by in-plane and out-of-plane actions:

- gravity actions, related to the normal service of the structure
- seismic actions, related to inertial forces due to the active masses

While masonry buildings feature a good resistance with respect to gravity loads (permanent and variable) being stressed mainly in compression, they do suffer under seismic loads since shear and tensile strengths are rather poor.

In seismic analysis, the stress and strain states due to horizontal actions are particularly important.

Under in-plane shear two different resistant mechanisms may occur:

- sliding shear: the strength is related to the friction and it increase with the normal stress
- diagonal shear: the strength is related to a conventional tensile strength

The flexure resistance is related to the compressive strength and to the stress distribution. In fact, since the tensile strength of masonry is null, only a part of the cross section reacts to flexure.

The seismic resistant mechanisms depend on several factors: geometry of the wall, quality of the material, restraint conditions, applied loads.

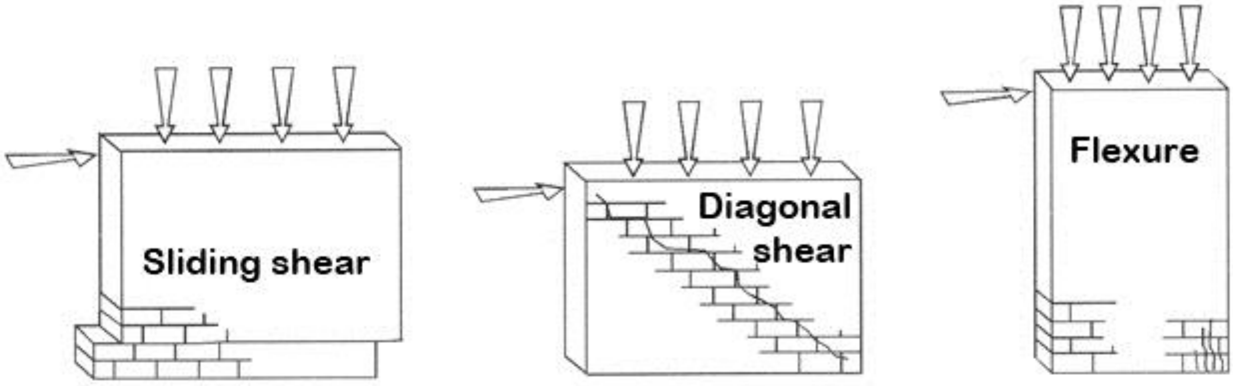

Fig. 1.1.1. Resistant mechanism of masonry walls [1]

In general the wall may develop either shear or flexure mechanisms.

In case of low vertical load and poor quality mortar, the seismic loads frequently cause sliding of the upper part of the wall along one of the mortar joints, provided that masonry is regular and features horizontal mortar joints (as in brick masonry). This mechanism is referred to as sliding shear.

Mechanisms of diagonal shear, typical of unreinforced masonry under seismic load, occur when the principal tensile stress overcome the resistance of the material. When the wall reaches its ultimate strength against lateral loads, it develops diagonal cracks forming the characteristic X pattern due to the alternating direction of the seismic action (Fig. 1.1.2). The cracks may develop along the mortar joints or even through the masonry elements.

Fig. 1.1.2. Diagonal shear cracks [2]

Finally, in case of good shear resistance and high moment to shear ratio, the wall may develop a flexure mechanism showing collapse of the elements in the compressive area at the edge of the cross section.

In order to characterize a masonry material, the following mechanical properties must be defined:

- Modulus of elasticity E
- Shear Modulus G

For new masonry:

- Characteristic compressive strength f_k
- Characteristic initial shear strength f_{vk0}

For existing masonry:

- Mean compressive strength f_m
- Mean initial shear strength τ_0

The structural analysis of a masonry building is performed with the following step:

1. Definition of the analysis object: new or existing building. In case of new building, the architectural project provides all the information about geometry, materials and construction details. In case of existing building the knowledge of the structures is achieved through geometric and visual survey, in-situ and laboratory testing.
2. Identification of primary and secondary structural elements
3. Structural modelling
4. Load definition
5. Structural analysis
6. Safety verifications
7. If the verifications are not fulfilled: in case of new building the design must be partially or totally revised; in case of existing building it may be considered the application of strengthening measures.

The structural system of a masonry building consists of walls and slabs, both two-dimensional elements. Under vertical loads slabs undergo out-of-plane actions while walls undergo in-plane and out-of-plane actions considering eventual misalignment of the overlying walls and the eccentricity of the load transferred by the slabs. Under horizontal loads, slabs collect and distribute the action to the walls which must ensure the resistance of the building.

A structural system like the one in Fig. 1.1.3 develops the typical box behavior, if the following conditions are met:

- slabs are well connected to all the boundary walls
- slabs can be assumed infinitely rigid under in-plane actions
- all the walls are well connected to each other

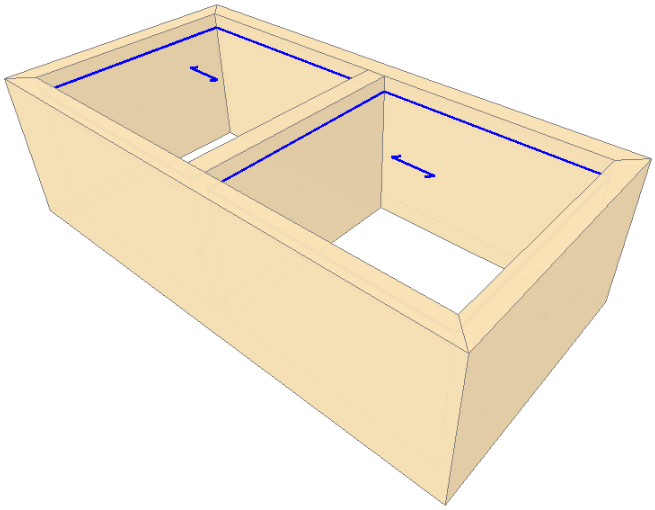

Fig. 1.1.3. Box behavior of masonry structure

In the ideal box behavior, under seismic action:

- seismic horizontal inertial forces act on the slab due to the total mass (structure self-weight, permanent and variable applied loads)
- the slab behaves rigidly and distributes the horizontal forces to the walls, parallel and orthogonal to the seismic action
- the walls undergo in-plane and out-of-plane actions, since the seismic action is along a generic direction

The analysis of masonry buildings requires to: (a) identify the effective seismic resistant vertical elements, masonry piers; (b) consider the contribution of spandrels and parapets; (c) correctly interpret the structural scheme with definition of restraints and loads; (d) characterize masonry material through adequate mechanical parameters; (e) assess the local response of the structure checking the quality of the connections.

1.2 SEISMIC ANALYSIS: FIRST AND SECOND DAMAGE MODE

Damage to masonry buildings can be interpreted on the basis of two fundamental modes:

- **First Damage Mode**, produced by seismic actions perpendicular to the wall that cause overturning, out-of-plane flexure, failure of connections. These mechanisms are referred to as first damage mode since they are the ones more likely to occur in existing buildings under seismic action. In new buildings the modern construction techniques and the good quality of the connections generally prevent the occurrence of these mechanisms.
- **Second Damage Mode,** produced by horizontal forces acting in the plane of the wall.

In order to prevent damages of the first mode, the analysis of the local collapse mechanisms allows to check the stability of several structural portions called macroelements. A typical case is the out-of-plane behavior of a wall, which should be assessed with respect to simple overturning and horizontal or vertical flexure with the development of intermediate hinges. The analysis yields the value of the collapse multiplier, that correspond to the maximum seismic action the macroelement can withstand. Generally applied to existing buildings, the analysis aims to assess the seismic improvement of the building after strengthening measures. The collapse multipliers after interventions should be higher than the ones before interventions.

Fig. 1.2.1. First damage mode: collapse mechanisms [3]

Once that local collapse mechanisms have been prevented, the overall behavior of the building and its safety with respect to second mode damages should be assessed through global analysis. This analysis assumes that the walls are well connected with each other and that the building behaves as a whole. The analysis can follow static nonlinear procedures: simplified as the POR method or general as Pushover analysis. The next paragraphs deal with the global analysis of buildings.

1.3 IN-PLANE BEHAVIOR OF A SINGLE MASONRY WALL

In a masonry wall the piers are the main resistant elements. A wall under in-plane horizontal actions can be considered as a set of several piers working in parallel (Fig. 1.3.1).

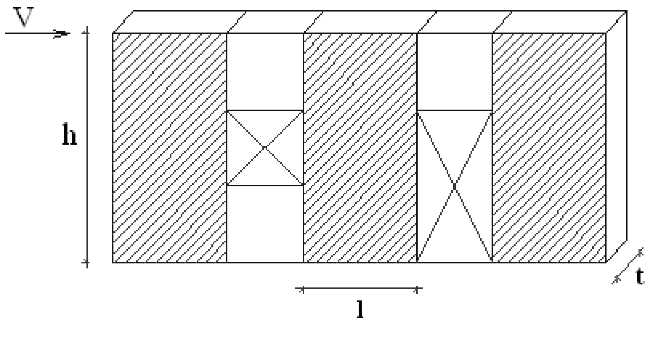

Fig. 1.3.1

Let us assume masonry piers schematized as one-dimensional elements, fixed at the base and connected at the top through a "deep beam". Under a horizontal force, the piers deform differently whether the top of the wall is flexible or rigid. This results in different values of the flexural stiffness (Fig. 1.3.2): if the top of the wall is flexible the pier behave as a cantilever (n=3), if it is rigid the top end of the pier is fixed (n=12).

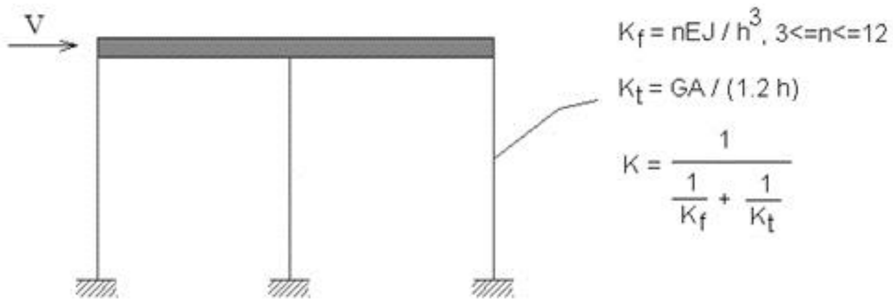

Fig. 1.3.2

Let us consider a single pier under the action of a horizontal force V (Fig. 1.3.3).

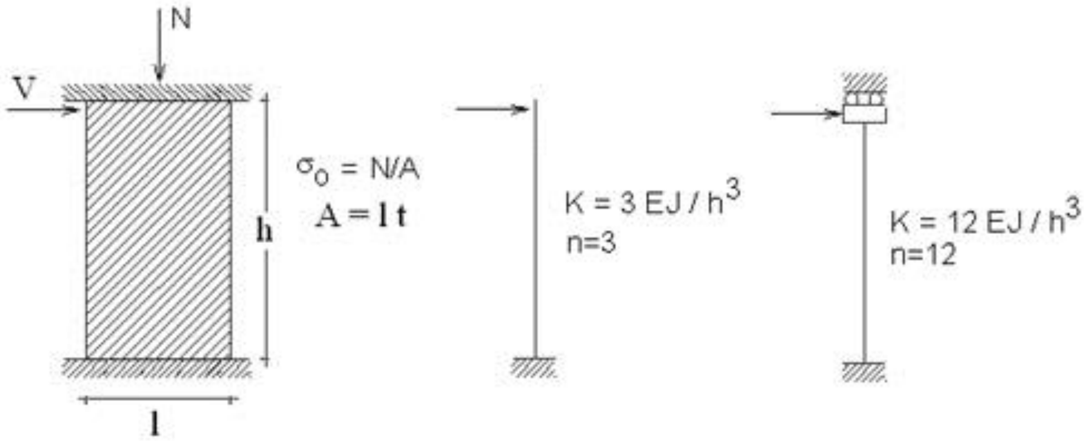

Fig. 1.3.3.

Combining flexural and shear deformation, the top displacement is given by:

$$\delta = \delta_f + \delta_t = \frac{Vh^3}{n\,EJ} + \chi\frac{Vh}{GA}\,, \qquad \chi = 1.2,\; 3 \le n \le 12 \tag{1}$$

Setting $\delta{=}1$, the translational stiffness is:

$$K = \frac{1}{\frac{h^3}{n\,EJ} + \frac{1.2h}{GA}} \tag{2}$$

The elastic translational stiffness is one of the three parameters required to define the structural behavior of a masonry pier, which can be assumed elastic-perfectly plastic. In fact, the constitutive law is described by the force-displacement diagram in Fig. 1.3.4 and the following parameters:

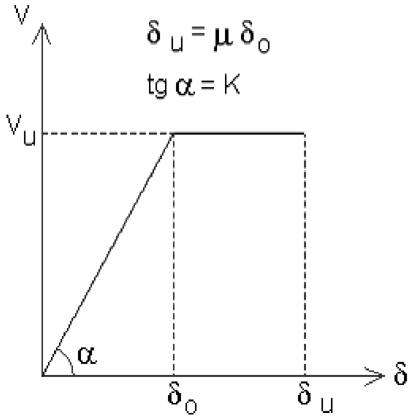

Fig. 1.3.4.

- Elastic stiffness K, expressed by Equation (2), which provides the slope of the elastic branch
- Ultimate force V_u which following the traditional methodologies (POR method) represents the ultimate diagonal shear resistance of the pier:

$$V_u = A \cdot \frac{f_{td}}{b} \sqrt{1 + \frac{\sigma_0}{f_{td}}} \tag{3}$$

  where: A is the area of the pier cross section, σ_0 is the average normal stress, $f_{td} = 1.5\, \tau_{0d}$ is the design values of the tensile strength for diagonal cracking and τ_{0d} is the reference shear strength of masonry, b is a correction factor dependent on the slenderness of the wall ($b = h/l\, , with\, 1 \leq b \leq 1.5$)

- The ductility μ, equal to the ratio of ultimate to elastic displacement, traditionally taken equal to 1.5 for existing masonry and 2 for new masonry. In recent methodologies, the ductility approach has been replaced by defining the ultimate displacement as a fraction of the pier height. However, both approaches aim to describe adequately the plastic branch of the pier behavior.

1.4 IN-PLANE BEHAVIOR OF MULTIPLE PIERS MASONRY WALL

The structural behavior of a multiple piers wall is defined starting from the behavior of the single pier. Under the action of a horizontal force all piers show the same top displacement but each one reacts with a force related to its translational stiffness. Therefore, the force-displacement diagram of the wall is given by the sum of the contributions of each pier (Fig. 1.4.1).

From the global force-displacement diagram, given the value of the horizontal force, it is possible to derive the corresponding displacement of the wall, while the reaction of each pier can be read on the respective diagram.

For instance, let us consider the case in Fig. 1.4.1 related to a three piers wall. Let us suppose that the force-displacement diagrams of the piers are those shown in the figure and that we want to plot the diagram that describes the overall behavior of the wall.

Up to point A the overall behavior of the wall is the sum of the elastic contributions of the three piers. Point A marks the end of the elastic phase since pier 3 reaches its elastic limit. Point B and point C mark the achievement of the elastic limit respectively for pier 1 and pier 2.

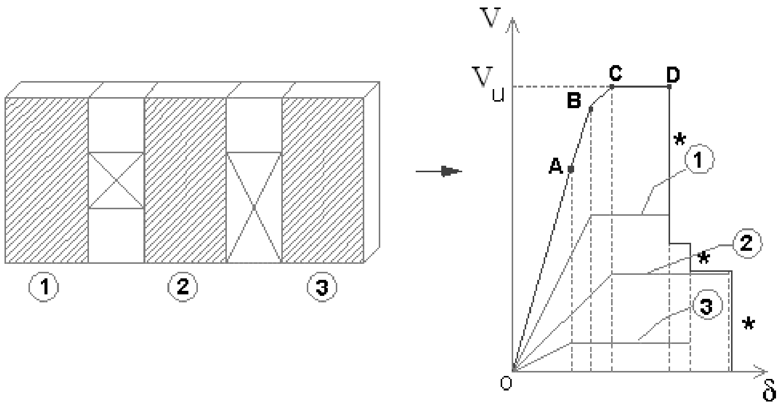

Fig. 1.4.1. Global force-displacement diagram

After point C, the diagram features a horizontal branch corresponding to the ultimate resistance of the wall. The horizontal branch ends in D where pier 1 contribution goes down. Further on, the diagram features several steps corresponding to the subsequent resistance falls of the other two piers. The global force-displacement diagram describes the nonlinear behavior of the wall.

1.5 A SPATIAL MASONRY SYSTEM: THE POR METHOD

For a given story, a curve like the one in Fig. 1.4.1 could be easily obtained taking into account the contributions of the piers of all the walls. This procedure, however, would apply only if the center of stiffness coincides with the center of mass of the story, since it only considers the horizontal translation of the piers and no torsional effects. In the general case, the seismic force is applied in the center of mass and produces not only translation but also rotation around the center of stiffness. Therefore, in order to plot the capacity curve of the story as a whole, the algorithm must take into account also the torsional effects: the POR method [4] complies with this requirement.

Initially, for each story, the positions of center of mass and center of stiffness, the eccentricity and the polar moment are determined. Then the analysis is performed separately in the X and Y direction, according to the following procedure.

The minimum displacement of the center of stiffness leading a pier to the elastic limit marks the end of the elastic behavior of the system. Up to this stage the stiffness of each pier is elastic. Starting from the elastic limit state, an incremental process allows to determine the resistance capacity of the system until cracking and ultimate limit state. The procedure catches the behavior of the structure step by step: conventionally, the cracking limit state is reached when the displacement of one pier reaches 1.2 times its elastic limit; whereas, the ultimate limit state is reached when at the next step of the analysis displacement increases with a stationary or decreasing reaction force.

The safety verification is satisfied if the reactive force of the story is higher than the seismic shear that it should sustain, that is the sum of the seismic forces acting above the story.

Of course, the procedure must take into account the plastic behavior of the piers that exceed the elastic limit. This can be done by replacing the elastic stiffness K_0 with the secant stiffness $K(\delta)$, expressed by the following equations and shown in Fig. 1.5.1.

$$
\begin{aligned}
K = K_0 = \tan\alpha_0 = \frac{V_u}{\delta_0}, \qquad & \delta \le \delta_0 \\
K = K(\delta) = \tan\alpha = \frac{V_u}{\delta}, \quad & \delta_0 < \delta \le \delta_0
\end{aligned}
\tag{4}
$$

Therefore, after the elastic limit, the pier stiffness depends on its displacements while the reaction remain constantly equal to V_u.

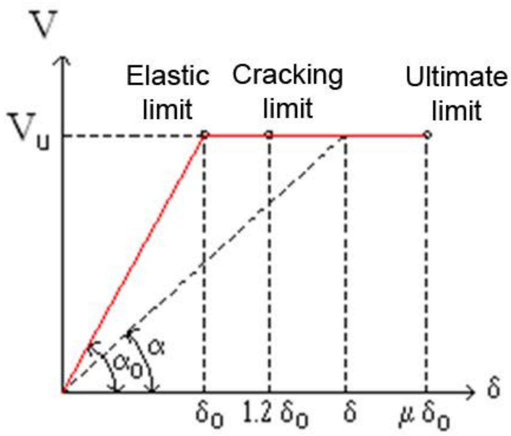

Fig. 1.5.1. Shear-displacement diagram of a single pier

At each step of the procedure the position of the center of stiffness must be updated taking into account the current stiffness values calculated with Equation (4).

The diagram shear-displacement V-δ is used to describe the behavior of a given story of the building under seismic action in X or Y direction. The diagram, that for a spatial model corresponds to the curve in Fig. 1.4.1, is obtained step-by-step plotting the horizontal displacement of the center of mass (δ) against the total reacting force developed by the walls (V). (Fig. 1.5.2)

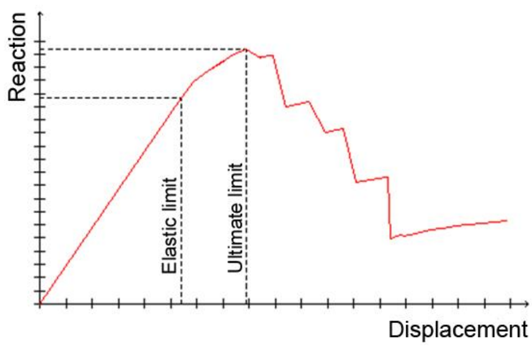

Fig. 1.5.2. Shear-Displacement diagram

The POR method is actually a spatial static nonlinear analysis based on the following hypotheses:

1. The analysis is performed separately story by story
2. The ultimate shear is determined on the basis of the diagonal shear with no reference to sliding shear or flexure
3. The piers are restrained as in a shear-type frame, thus they feature fixed-fixed ends
4. In each pier the axial force is assumed constant and equal to the static value with no variation due to the seismic action
5. The piers feature a bilinear behavior elastic-perfectly plastic
6. The slabs are assumed infinitely rigid in their plane so that the horizontal force is distributed to the piers depending on their stiffness
7. The piers feature constant cross section along the height of the story
8. Seismic action is only applied at the top of the story

For these hypotheses the POR method can be successfully applied only to limited typologies of buildings, featuring stubby piers with prevalent shear behavior, rigid slabs, low number of stories. Therefore, it is necessary to define an extended method able to overcome the most restrictive limitations. In particular, the new method should intervene on the first four hypotheses:

1. It should allow to analyze the building as a whole instead of story-by-story.
2. Beside the diagonal shear mechanism, it should account for in-plane and out-of-plane flexure as well as sliding shear. Moreover, piers under tensile axial force should be considered nonreactive.
3. In some cases, it should allow the rotation of the nodes at the base and at the top of masonry piers (no shear-type)
4. It should allow variation of the pier axial force due to seismic action.

The following paragraphs introduce the Equivalent Frame Model and describe a static nonlinear procedure that can be considered as the extension of the POR method.

1.6 THE EQUIVALENT FRAME MODEL

The finite element method constitutes the basis of all the modern methods of structural analysis. Even the POR method, described above, is an application of the finite elements method: the piers are frame elements fixed at the base and connected at the top through infinitely rigid beams.

In general, a masonry building can be modelled as a set of one-dimensional elements, vertical and horizontal, connected in the joints. In this way, it would appear as a finite element model very similar to the modelling of steel and RC frames.

Adopting one-dimensional elements leads to several advantages. Although masonry buildings are made of walls which could be treated as two-dimensional elements, 2D modelling demands a rather high computational effort [11] and requires complex algorithms in order to account for the nonlinear behavior and the lack of tensile strength of the material. A simplified model implements these features much more easily and provides directly the values of the internal forces (axial force, shear, bending moments) which are needed for the safety verifications. But the most important remark is the illusion that more complex models leads to more accurate results. The main issue in the analysis of masonry buildings is the knowledge of all the involved parameters such as mechanical characteristics of the materials, construction details, restraints and loads; therefore, especially for existing buildings, the use of complex models combined with uncertain parameters can be very misleading.

For all these reasons, the Equivalent Frame Model is well-regarded by the recent Standards and is the basis for the definition of fully comprehensible methods for nonlinear analysis of masonry buildings [5, 12].

1.6.1 About the finite element method

The finite element method (FEM) is the generalization of the matrix displacement method. The basic concept of FEM is modelling of the structure as a set of discrete elements connected through joints.

Through the equilibrium relations of each joint, the stiffness matrixes of each element are assembled into a single system of linear equations. The solution of the system, the vector of joint displacements, leads to deformations and internal forces of each single element. The joint displacements (translational and rotational) represents the degrees of freedom of the structure.

The method can be summarized as follows:

- Modelling of the structure through joints and elements
- Definition of stiffness matrix of each element
- Assembly of the global stiffness matrix and solution of the linear system: Ku=F, where K is the global stiffness matrix, u is the vector of the joint displacement and F is the vector of the known joint external forces.

Since the method derive from the displacement method, the more restrained is the structure the lower is the number of the unknowns. Buildings with rigid levels feature a lower number of degrees of freedom since, for a given level, the only independent horizontal displacement is the one of the "master" joint. The horizontal displacement of the other joints of the same level, the "slave" joints depends on the one of the master joint.

The formulation Ku=F is typical of the linear systems. Linear analyses, represents the first class of analyses used in seismic engineering: equivalent static analysis (lateral force method of analysis) and dynamic modal analysis (modal response spectrum analysis).

In equivalent static analysis the seismic forces are distributed throughout the structure according to a triangular pattern that approximates the shape of the fundamental mode of vibration. The force applied to each joint is proportional to its mass and height.

In dynamic modal analysis, a modal analysis of the structure is performed before the definition of the seismic forces. The free vibration of a linear elastic system can be considered as the superposition of many simple vibrations correspondent to different modal shape and period of vibration. The modes of vibration (shape and period) depend on stiffness and mass of the system and can be calculated solving the eigenvalues problem. The seismic forces applied to the structure are calculated taking into account the fundamental modes of vibration and a number of modes in order to reach a high value of the total participating mass (85%). Of course, in case of spatial analysis, the modes of vibration can be along the X and Y direction or torsional when the centers of stiffness and mass are not coincident. Once the seismic force have been defined, the procedure of dynamic modal analysis is similar to the one for equivalent static analysis.

The seismic forces are inertial, therefore, for both equivalent static analysis and dynamic modal analysis, they can be calculated knowing the structural acceleration. The design response spectra provide the structural acceleration based on several factors: period of vibration (approximated or calculated through modal analysis), ground acceleration, soil characteristics.

The main characteristic of linear analysis is the hypothesis that the behavior of the structure is linear elastic. If the expected behavior is nonlinear, that is the displacements are not always proportional to the applied forces, a nonlinear analysis is necessary.

Under the action of increasing forces, some parts of the structure may behave plastically, collapse or become nonreactive. Therefore, the internal releases and consequently the stiffness of the elements must be progressively updated in order to correctly evaluate the response of the structure.

The system to be solved become $K(u)\ u = F$, where the stiffness matrix depends on the displacement. The nonlinearity does not allow the solution of the problem in just one step but requires an incremental procedure where at each step the element stiffness can be updated following the real behavior of the structure. The force-displacement diagram is independent form the seismic demand and can be considered an intrinsic characteristic of the structure bases on its resistance capacity, therefore it is referred to as capacity curve. Once the capacity curve has been created, it allows the safety verification based on a comparison with the seismic demand.

1.6.2 Masonry requires nonlinear analysis

In previous paragraphs we saw that the structural behavior of masonry piers is nonlinear (elastic-perfectly plastic behavior). Shear or flexure failure may void the resistance of the element as well as the occurrence of tensile axial forces makes it nonreactive. For these reasons, linear analyses are not able to catch the real behavior of the structure.

Moreover, the safety verifications applied at the end of linear analyses can be misleading: the presence of one element that does not fulfill the verifications lower the overall seismic capacity of the structure since the latter is identified as the capacity of the weaker elements.

Therefore, the behavior of masonry structures can be properly assessed only through nonlinear analysis, in fact even the simplified POR method was nonlinear.

1.6.3 Equivalent frame model

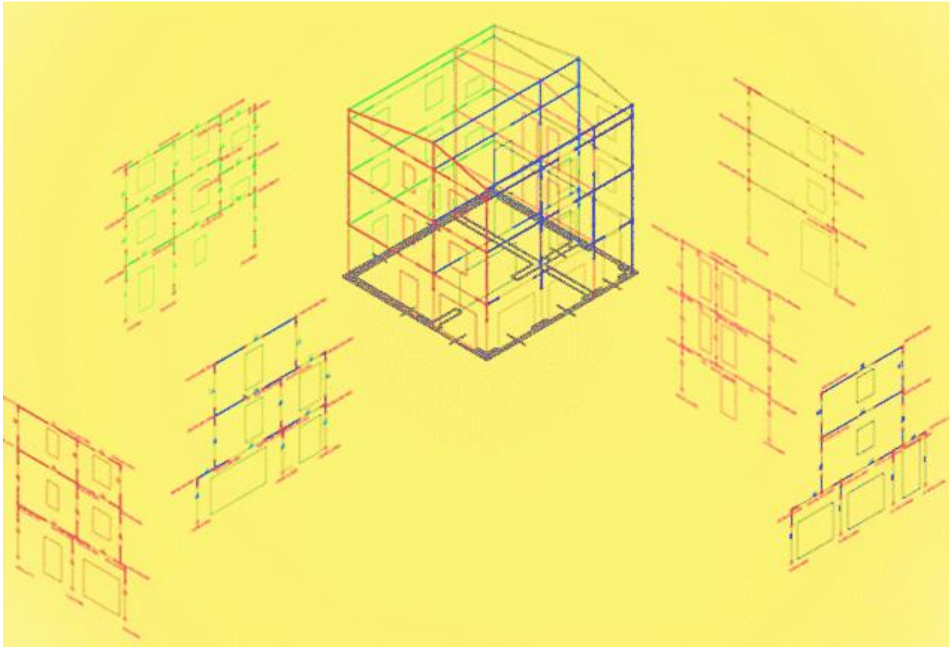

Fig. 1.6.1. Equivalent frame model

Fig. 1.6.1 shows a masonry building modelled through the equivalent frame method, the model can be considered as the assembly of several plane frames corresponding to the different walls. The vertical frame elements are the masonry piers while the horizontal elements correspond to spandrels and parapets.

This structural model represents adequately the effective mass and stiffness distribution. In general, a masonry building is analyzed through a spatial model, but planar models may be used for the analysis of regular buildings along the two main horizontal directions or for buildings with non-rigid slabs where each wall behaves independently from the structure as a whole.

Fig. 1.6.2 shows the model of a single wall (the façade of the building) and highlights some of the characteristics of the equivalent frame modelling. The model features rigid offsets at the ends of piers and spandrels. The rigid offsets have different lengths for in-plane and out-of-plane flexure, in fact for in-plane flexure they correspond to the area of intersection between piers and spandrels while for out-of-plane flexure they correspond to the thickness of the slabs which reduce the free span of the pier.

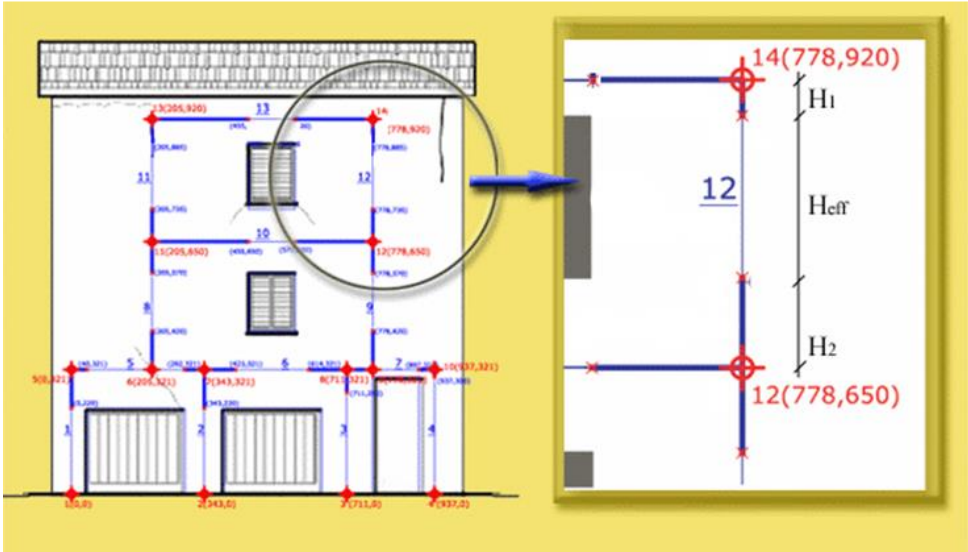

Fig. 1.6.2

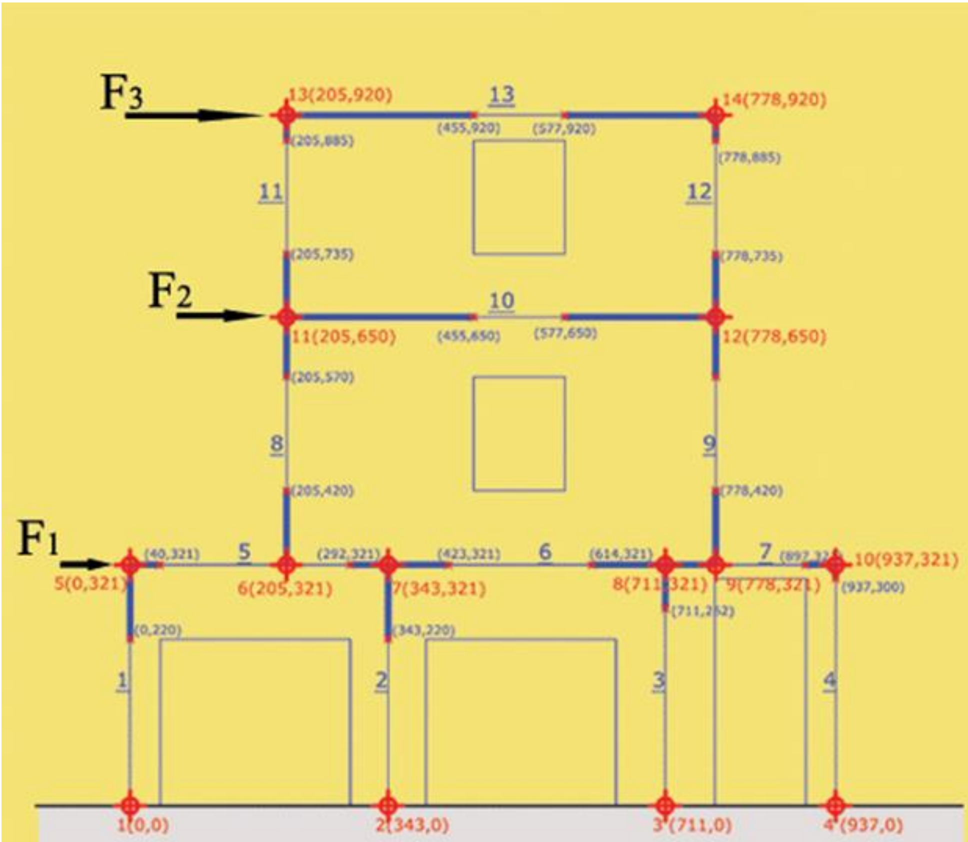

Fig. 1.6.3

Fig. 1.6.3 shows in more detail the structural scheme giving the ID of frames and joints and highlighting rigid offsets and rigid links. The walls can be considered fixed at the base or supported by Winkler foundation beams defined rigid below the piers and deformable below the openings.

Fig. 1.6.4 highlights some characteristic of the spatial modelling. The intersection of orthogonal walls is achieved through rigid links located at the top of the walls.

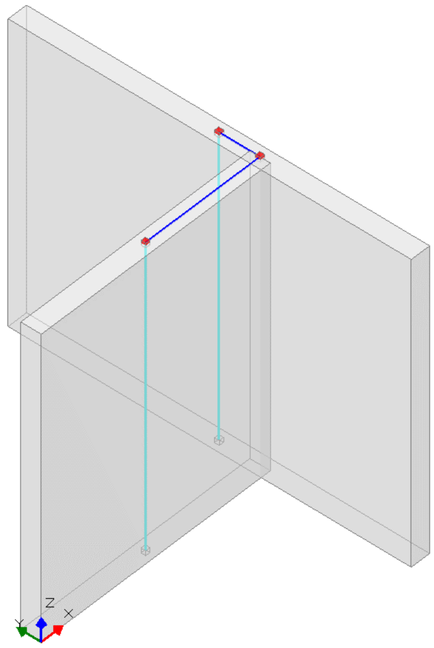

Fig. 1.6.4. Orthogonal walls connection

1.6.4 Nonlinear static analysis (Pushover)

The concept at the base of nonlinear static analysis is that the seismic capacity of the structure can be described by its behavior under a system of increasing static forces. The system of horizontal forces must simulate in the best possible way the inertial forces produced by the seismic action in the horizontal direction.

Masonry elements feature an elastic-perfectly plastic bilinear behavior with constant tangential stiffness in the elastic range and zero tangential stiffness in the plastic range [6].

The analysis is performed through a series of linear static analyses where the structural model is constantly updated in order to account for the stiffness modifications (e.g. stiffness reduction of the elements that enter the plastic range).

1.6.4.1 Capacity curve

The capacity of the structure is represented by the capacity curve, which is a plot of the total base shear against the displacement of the control point (normally taken as the center of mass of the last story). The capacity curve is an intrinsic characteristic of the structure. It allows the safety verification of the structure through the definition of the equivalent single-degree-of-freedom curve (SDOF) and its comparison with the seismic demand represented by the response spectrum. The seismic demand can be also considered as a displacement demand, which represents the target displacement that the structure should be able to sustain according to a given seismic action.

1.6.4.2 Pushover algorithm

The analysis is performed along a given direction and for a given distribution of the lateral loads. A proper base shear increment is chosen, suggested values range from 1/50 to 1/10 of the maximum base shear.

1. The structure subjected only to vertical loads is analyzed.
2. The base shear increment is applied, the lateral forces are distributed to the structure according to the chosen load pattern.
3. The internal actions of the structural elements under the combination of vertical and lateral loads are calculated. At each step of the analysis the incremental internal actions and displacements are added to the corresponding values of the previous step.
4. The total base shear and the control displacement are calculated, their values represent one of the points of the capacity curve.
5. The following safety verifications are applied to the structural elements: in-plane flexure, sliding shear, diagonal cracking shear, tensile axial force, out-of-plane flexure (for out-of-plane flexure the element features an elastic-brittle behavior with no plastic range). If all the verifications are satisfied, the internal releases of the element remain the same. When flexure or shear verifications are not satisfied the element enters the plastic range and plastic hinges are inserted in the model in order to account for the new behavior of the element which deforms under constant internal actions.

   If shear (sliding or diagonal) reaches the maximum value, it has to remain constant during the next incremental steps: the secant shear stiffness will progressively decrease while the tangent shear stiffness becomes null. In order to account for this behavior the element ends turn into pinned-pinned. In this way in the next steps of the analysis the shear acting in the frame will remain constant. The in-plane flexure verification will still be applied checking whether the variation of axial force leads to an exceedance of compressive or tensile strength.

   If in-plane flexure verification is not satisfied at one of the end sections of the element, a plastic hinge is defined in that section. Even in this case while the tangent rotational stiffness becomes null, the secant stiffness progressively decrease. After the insertion of the plastic hinge the incremental bending moment is null and the total bending moment in the given section remains constant.

   Therefore, if one or more verifications are not satisfied the model is modified and the stiffness matrix is updated according to the new internal releases of the elements. In case of rigid offsets at the ends of the frame, the plastic hinges are defined at the ends of the deformable span.

   If the distribution of the lateral loads is proportional to the modal shape and adaptive, that is it follows the dynamic characteristics of the structure, the pattern must be updated whenever the model is modified. In other words, the modification of the structural model leads to new modes of vibration and therefore to new distribution of lateral loads.
6. The steps 2, 3, 4, 5 are repeated until one of the piers reaches one of the following collapse limit states:
   - excessive in-plane deformation
   - the element is nonreactive due tensile stress;

- the element reached the maximum resistance with respect to out-of-plane flexure.

In this way a capacity curve like the one shown in Fig. 1.6.5(a) is drawn.

Fig. 1.6.5. Capacity curves

7. When one or more piers reach a collapse ultimate state the resistance of the structure drops. The incremental procedure cannot continue on the same curve because the redistribution of internal forces is not predictable. Therefore, the model is updated accounting for plastic and collapsed elements and the procedure starts again from step 1 with the generation of another sub-curve like the ones in Fig. 1.6.5(b).

   At the beginning of each curve, the structural model is updated as follows:

   - The elements that entered the plastic range for shear but did not yet collapse are assigned a reduced translational stiffness calculated as the secant stiffness at the end of the previous curve
   - The sections that in the previous curve developed a plastic hinges are assigned a reduced rotational stiffness through the insertion of a rotational spring with constant K given from the ratio between ultimate moment and plastic rotation (total rotation of the section minus the elastic rotation)
   - The elements collapsed for flexure or shear are assigned pinned-pinned ends. The hinges are in-plane or out-of-plane depending on the mechanism that caused collapse (Fig. 1.6.6b)
   - The axial force is released for the elements that developed tensile deformations, so that the element is unloaded (Fig. 1.6.6c)

   Therefore, the plastic hinges that interpret the elastic – plastic behavior of the elements are modelled as hinges (moment releases) during the creation of the curve and with stiffness reduction at the beginning of the next sub-curve.

   All the releases are applied at the ends of the deformable span. Since the length of the rigid offsets is generally different for the in-plane and out-of-plane behavior, the end releases may be applied in different sections accordingly.

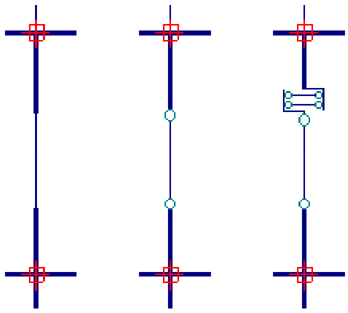

Fig. 1.6.6. Model update: end releases

8. The procedure stops when the structure becomes a mechanism or for an excessive value of the control displacement.
9. The final capacity curve that account for the drops of resistance of the structure is obtained connecting the sub-curves with vertical segments and assumes the characteristic sawtooth shape as shown in Fig. 1.6.5(c). This curve resemble the one obtained with the POR method, in fact they both represent the capacity curve of the structure.

1.6.4.3 Verifcations of masonry elements

Similarly to the POR method, the in-plane behavior of a masonry pier is assumed elastic – perfectly plastic (Fig. 1.3.4). However, contrary to the POR method where the axial force remains constant, the displacement at the elastic limit (δ_0) is not known a priori. In fact, the axial force and thus the shear resistance may vary during the analysis; therefore, δ_0 must be defined when the element reaches the ultimate shear capacity. The ultimate displacement can be defined based on the ductility of the element or as a percentage of the deformable span (0.4% in case of shear failure, 0.6% in case of flexure failure).
The ultimate shear capacity is the minimum between the capacity with respect to Diagonal shear (V_t) and Sliding shear (V_s) mechanism:

$$V_u = \min(V_t, V_s)$$

The capacity with respect to Diagonal shear mechanism is given by the following equation:

$$V_t = l \cdot t \cdot \frac{f_{td}}{b} \cdot \sqrt{1 + \frac{\sigma_0}{f_{td}}}$$

Where:

- l and t are the dimensions of the cross section;
- $f_{td} = b \cdot \tau_{0d}$ is the design tensile strength;
- τ_{0d} is the design initial shear strength. For existing masonry $\tau_{0d} = \tau_0 / CF$, where τ_0 is the mean initial shear strength and CF is the confidence factor. For new masonry $\tau_{0d} = f_{vm0} = 0.7 \cdot f_{vk0}$, where f_{vm0} and f_{vk0} are respectively the mean and characteristic value of the initial shear strength. Please note that in pushover analysis the material partial safety factor γ_m does not apply in the calculation the design values.

- b is a corrective coefficient related to stress distribution along the section and based on the slenderness λ of the element, $b = \lambda$ and $1 \leq b \leq 1.5$.

The capacity with respect to Sliding shear mechanism is given by the following equation:

$$V_t = l' \cdot t \cdot f_{vd}$$

Where:

- l' is the width of the compressed area of the section (reactive area) and t is its height
- f_{vd} is the design shear strength. For existing masonry $f_{vd} = (\tau_0 + 0.4\,\sigma_n)/CF$, while for new masonry $f_{vd} = f_{vm} = f_{vm0} + 0.4\,\sigma_n$. σ_n is the mean compressive stress in the given cross section

As far as regard the in-plane flexure, the relation moment – rotation is also elastic – perfectly plastic.

The ultimate moment is given by:

$$M_u = \frac{l^2\,t\,\sigma_0}{2}\left(1 - \frac{\sigma_0}{0.85\,f_d}\right)$$

Where:

- l is the width of the cross section
- t is the thickness of the wall
- σ_0 is the mean vertical stress given by $\sigma_0 = N/(l\,t)$, where N is the axial force
- f_d is the design compressive strength. For existing masonry $f_d = f_m/CF$ where f_m is the mean compressive strength. For new masonry $f_d = f_m$.

Besides the resistance verifications, masonry piers under tensile deformation are considered nonreactive with zero stiffness and zero resistance.

In spatial models, out-of-plane flexure verifications are also applied considering a behavior similar to the one for in-plane action but without plastic branch (elastic – brittle behavior). The ultimate moment for out-of-plane actions is calculated assuming a rectangular stress distribution with resistance equal to 0.85 f_d and ignoring the tensile strength. The following equations and figure show the calculation of the ultimate moment given the value of the ultimate axial force N_u:

$$N_u = l \cdot t \cdot 0.85 \cdot f_d$$

$$N = 2 \cdot u \cdot l \cdot 0.85 \cdot f_d \Rightarrow u = \frac{N}{0.85 \cdot f_d \cdot 2 \cdot l}$$

$$e = \frac{t}{2} - u, \quad A = l \cdot t$$

$$M = N \cdot e = N\left(\frac{t}{2} - u\right) = N\left(\frac{t}{2} - \frac{N}{0.85 \cdot f_d \cdot 2 \cdot l}\right) = N\frac{t}{2}\left(1 - \frac{N}{0.85 \cdot f_d \cdot A}\right) = N\frac{t}{2}\left(1 - \frac{N}{N_u}\right)$$

$$N = N_u \Longrightarrow M = 0, \quad N = 0 \Longrightarrow M = 0, \quad N = \frac{N_u}{2} \Longrightarrow M = N_u\frac{t}{8}$$

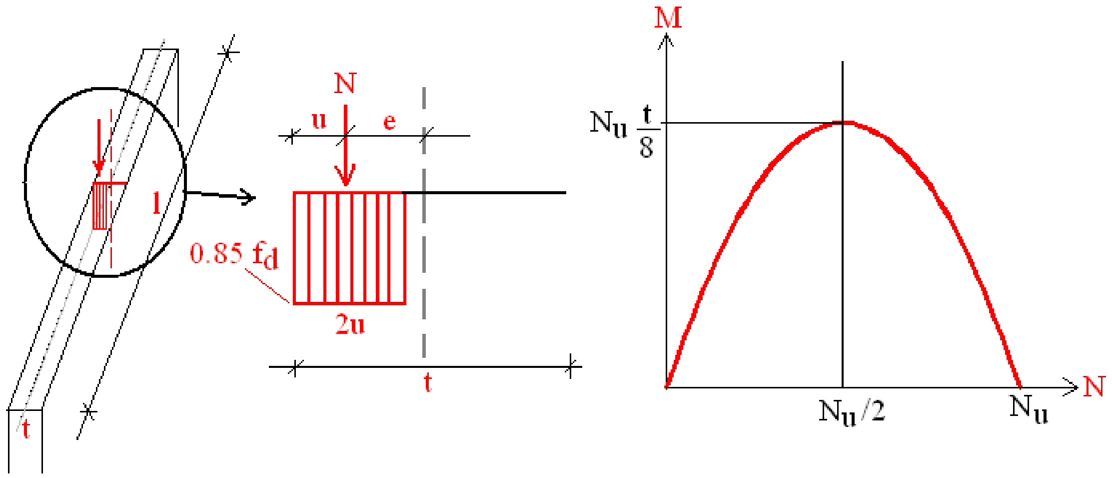

Fig. 1.6.7. Ultimate moment for out-of-plane flexure

So far, we covered exclusively masonry piers, but masonry buildings also feature horizontal masonry elements (spandrels) whose behavior is still the topic of several researches.

The paragraph [9, §7.8.2.2.4] introduces safety verifications for masonry spandrels in new buildings, demanding the application of shear and in-plane flexure verifications.

If the analysis have been performed with the hypothesis of rigid diaphragms, the axial force N in the spandrels is null: in this case the flexure verification can be applied if the spandrel features a tensile resistant element, such as a lintel. Then, the bending moment corresponds to a couple which leads to tensile stress in the lintel and compressive stress in masonry. If the value of the axial force N is not null, then the verification is similar to the one applied to masonry piers.

However, in many cases, the axial force N is not null but it is very low, therefore the in-plane flexure verification requires the presence of a tensile resistant element: the procedure is similar to the one applied for reinforced concrete or reinforced masonry elements.

As far as regards existing masonry buildings, the paragraph [10, §C8.7.1.4] specifies that in-plane flexure verification can be applied to masonry spandrels if they are able to develop a tensile strength (e.g. due to lintels, ring beams, tie rods, FRP strips). Anyhow, the assessment of the real capacity of masonry spandrels in existing buildings with respect to in-plane flexure still requires further investigations [5].

In fact, some models apply only shear verifications to masonry spandrels [7]. Since normally masonry spandrels do not feature continuous mortar joint in the vertical direction, the verification is not applied with respect to sliding shear but only for diagonal shear mechanism. The behavior of the spandrels can be assumed elastic – brittle or elastic – plastic, in the latter the ultimate displacement can be based on the failure of the adjacent masonry piers.

1.6.4.4 Force distributions

Nonlinear static analysis (pushover) is based on a system of static forces applied to each joint with mass or to the master joint of the level according to a given force distribution. During the analysis the forces increase but they retain the same proportion. Only in case of adaptive distributions, the proportion of the forces may change due to the updated modal characteristics of the structure. The analysis can be performed with the following force distributions.

Main distributions:

A) *Linear*: forces proportional to the ones used for equivalent static analysis (lateral force method)
B) *Unimodal*: forces proportional to masses and deformed shape of the fundamental mode of vibration.
C) *Dynamic*: forces proportional to the ones used for dynamic modal analysis (response spectrum)
D) *Multimodal*: forces proportional to masses and "equivalent" modal deformed shape which account for all the significant modes of vibrations. This distribution is proper for buildings with many stories or strongly irregular, where the effects of the next modes of vibration is very important. Usually masonry building are not that tall, still these distribution may be considered in case of irregular floor plans in order to account for eventual twisting behavior.

Secondary distributions:

E) *Uniform*: forces proportional to masses
F) *Adaptive Unimodal*
G) *Adaptive Dynamic*
H) *Adaptive Multimodal*

The adaptive distributions require the execution of modal analysis any time the stiffness of the structural elements changes (e.g. elements that enter the plastic range, elements that collapse, etc.). Therefore, based on the results of modal analysis the proportion between the different forces may change during the incremental analysis.

The following equations describe the formula used for the calculation of the incremental forces to be applied at each level for distribution A, B, E and D:

A) Linear $F_i = \frac{W_i z_i}{\sum_{j=1}^{n} W_j z_j} V$

B) Modal $F_i = \frac{W_i \Phi_{1i}}{\sum_{j=1}^{n} W_j \Phi_{1j}} V$

E) Uniform $F_i = \frac{W_i}{\sum_{j=1}^{n} W_j} V$

D) Multimodal $F_i = \frac{W_i \Phi_{eq,i}}{\sum_{j=1}^{n} W_j \Phi_{eq,j}} V \quad \Phi_{eq,i} = \sqrt{\sum_{k=1}^{m} (\Phi_{ki} \Gamma_k)^2}$

Where:

- F_i is the horizontal force applied to level i
- n is the number of levels
- V is the base shear increment
- W_i is the mass associated to level i
- z_i is the elevation of level i

- Φ_{1i} is the amplitude of the fundamental mode at level i
- $\Phi_{eq,i}$ is the amplitude of the equivalent mode at level i
- Φ_{ki} is the amplitude of mode k at level i
- Γ_k is the participation factor of mode k
- m is the number of considered modes

The following figure shows the example of a 4-storey masonry building and the load patterns for distributions A, B and E (the color legend is given in the above equations). Although the building is regular in plan and in elevation, the masses associated to the different levels are not equal: this is due to the way the lumped masses contribute to the total mass of the level (the mass of the last level is associated to less joint masses) and to the different load conditions at the different levels. In fact, if the masses were equal at the different level the distribution A would be triangular and the distribution E would be constant, but it is not the case of this example.

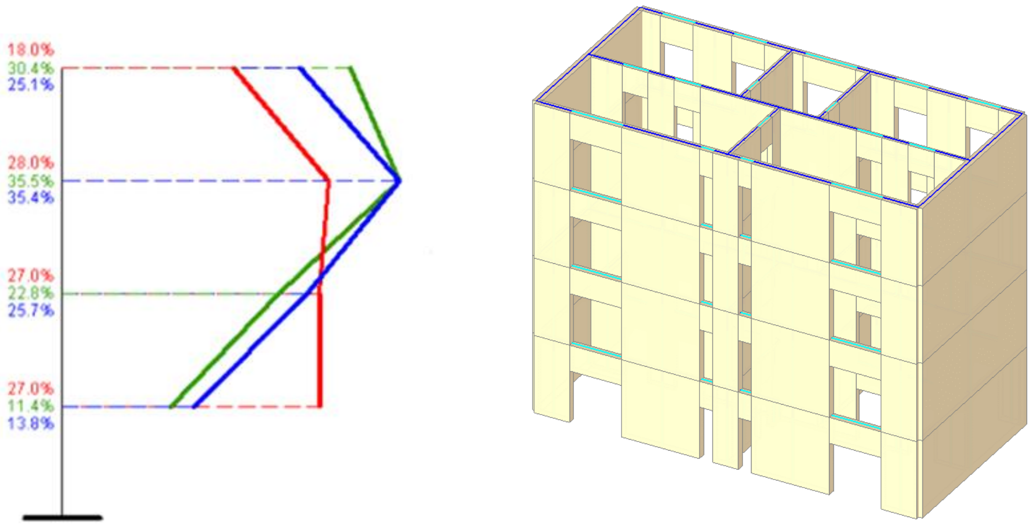

Fig. 1.6.8. Force distributions in Pushover Analysis

1.6.4.5 More about the differences with the POR Method

In case of a building with rigid diaphragms, Pushover Analysis correspond to the POR Method whether:

- In-plane flexure and sliding shear are disregarded (POR method applies verification only for diagonal shear)
- The building features only one story (POR method analyze the building story by story while pushover analysis is performed on the building as a whole)
- The model has shear-type behavior and feature one rigid level with master joint

However the following important difference about nonlinear procedure remains:

- In POR method incremental displacement is applied to the center of mass of the story and the safety verification is based on the comparison between the capacity in terms of resistant force and the seismic demand in terms of base shear.

- In Pushover analysis incremental forces are applied to the joints and the safety verification is based on a comparison between capacity and demand in terms of displacement

1.6.4.6 Safety verifications in Pushover Analysis: Ultimate Limit State

In Pushover Analysis several capacity curves are processed for the same building: along the two main horizontal direction X and Y; in the positive and negative directions (unless the building is symmetric); with or without the effects of accidental eccentricity; for several force distributions (according to [9, §7.3.4.1], at least the two distributions A and E). The analyses are performed based on the seismic load combination defined in [9, §3.2.4].

The safety verification described below must be applied to all the processed curves and the most severe condition will be used for the definition of the seismic capacity of the structure.

According to the Standards, the ultimate displacement capacity is taken as the displacement at which total lateral resistance (base shear) has dropped below 80% of the peak resistance of the structure, due to progressive damage and failure of lateral load resisting elements.

Once the displacement capacity as been defined it should be compared with the seismic displacement demand, defined with the following procedure [10, §C7.3.4.1]:

- Transformation to an equivalent Single Degree of Freedom (SDOF) system through the transformation factor Γ;
- Determination of the idealized elastic-perfectly plastic force-displacement relationship
- Determination of the period T* of the idealized equivalent SDOF system
- Determination of the target displacement for the equivalent SDOF system through elastic response spectrum

1.7 THE ANALYSIS WITH AEDES.PCM

The characteristics and the potentials of the Equivalent Frame Model have been widely studied by authoritative experts [5, 7] providing algorithms for the analysis and the assessment of masonry buildings. Based on these references and further research on this field, Aedes Software developed the Pushover Analysis on the Equivalent Frame Model a procedure that represents a significant step forward with respect to the traditional POR method.

Aedes.PCM creates the equivalent frame model of the building according to the prescription of the current Standards. The structural model consists of joints and frame elements and the analysis is performed according to the finite element method. The frame elements represent masonry piers and spandrels as well as columns and beams of other materials such as steel and reinforced concrete. The model features rigid links and rigid offsets for the proper description of the in-plane and out-of-plane behavior of the structure.

The frame end releases are fully customizable (e.g. slender piers may be represented as pinned-pinned frames) as well as the joint restraints. Master joints are used for the correct representation of rigid diaphragms. If the levels are non-rigid levels each joint mass is considered in the actual position and pushover analysis can still be performed: in this case the control displacement is assumed as the centroid of the last level in the deformed shape.

Compared to the equivalent frame model, the adoption of two-dimensional or three-dimensional finite elements mesh with the need of complex nonlinear constitutive laws (masonry nonlinearity is primarily due to no tensile strength) appears onerous and inconvenient. It represents a complex methodology, which can be hardly applied in case of irregular structures with non-aligned openings typical of existing buildings. Moreover, complex modelling, especially if not accompanied by precise estimations of the material behavior, leads to the illusion of more accurate results, while it is well known that the structural analysis of masonry building is anyhow affected by many uncertainties. For all these reasons, the adoption of simpler methodologies like the equivalent frame model is preferable.

REFERENCES

[1] M. Tomazevic, Verifica di resistenza sismica negli edifici in muratura: seguendo le nuove tendenze, *Murature Oggi*, no. 60, September 1998 (in Italian)

[2] A. Benedetti, Progettazione di Strutture in Muratura in Zona Sismica, Corso di aggiornamento sulle nuove Normative Sismiche, Imola, May 2004 (in Italian)

[3] M.R. Valluzzi, E. Michielon, C. Modena, L. Binda, Modellazione del comportamento di Edifici in Muratura sotto Azioni Sismiche: l'Esperienza Umbria-Marche, *X Congresso Nazionale "L'Ingegneria Sismica in Italia"*, Potenza-Matera, September 9-13, 2001 (in Italian)

[4] F. Pugi, *Progettazione di Costruzioni in Muratura*, Firenze: ALINEA Editrice, 2000 (in Italian)

[5] G. Magenes, D. Bolognini, C. Braggio (A cura di), *Metodi semplificati per l'analisi sismica non lineare di edifici in muratura*, Roma: CNR-Gruppo Nazionale per la Difesa dai Terremoti, 2000 (in Italian)

[6] G. C. Beolchini, G. Di Pasquale, L. Gizzarelli, *La valutazione delle prestazioni sismiche di strutture esistenti in cemento armato: indicazioni dalle Linee Guida NEHRP*, Roma, Dicembre 2002 (in Italian)

[7] G. Magenes, G.M. Calvi, Prospettive per la calibrazione di metodi semplificati per l'analisi sismica di pareti murarie, *Convegno Nazionale "La Meccanica delle Murature tra Teoria e Progetto"*, Messina, September 18-20, 1996 (in Italian)

[8] L. Petrini, R. Pinho, G.M. Calvi, *Criteri di Progettazione Antisismica degli Edifici*, Pavia: IUSS Press, 2004 (in Italian)

[9] D.M. 14 gennaio 2008. Nuove norme tecniche per le costruzioni, 2008 (in Italian)

[10] Circolare 2 febbraio 2009, n. 617. Istruzioni per l'applicazione delle "Nuove norme tecniche per le costruzioni" di cui al D.M. 14 gennaio 2008, 2009 (in Italian)

[11] P. Roca, C. Molins, R. Marì, Strength Capacity of Masonry Wall Structures by the Equivalent Frame Method, *J. Struct. Eng*, 131, 1601-1610, 2005

[12] Y. Belmouden, P. Lestuzzi, An Equivalent frame model for seismic analysis of masonry and reinforced concrete buildings, *Construction and Building Materials*, 23, 40-53, 2009

[13] G. Magenes, Earthquake resistant design of masonry structures: rules, backgrounds, latest findings, *8th International Masonry Conference*, Dresden, Germany, 2010