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NONLINEAR ANALYSIS AND SEISMIC STRENGTHENING OF MASONRY ARCHES: THE BLOCK-JOINT AND BLOCK-BLOCK FEM MODELS

Francesco Pugi[1], Alessio Francioso[2]

[1] AEDES Software
56028 San Miniato Basso, Pisa, Italy
info@aedes.it

[2] National Technical University of Athens
15780 Zografou, Athens, Greece
alessiofrancioso@gmail.com

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Abstract. *The stability of masonry arches plays a crucial role in the seismic assessment of historic and monumental buildings. The block-joint FEM model represents a major step forward with respect to the traditional rigid-brittle approach, allowing the assessment of the in-plane and out-of-plane arch behaviour and modelling of spatial systems such as cross vaults. Recent improvements of the method, among which the blocks modelled as curved frame elements and the pursuit of a static admissible configuration at the first step of the pushover analysis, are introduced. Moreover, the simpler block-block model that captures the behaviour of the arch through a lower number of finite elements is presented. The effects of passive and active strengthening measures such as CFRP sheets or prestress state are also considered in the models.*

1 INTRODUCTION

The structural analysis of masonry arches can be performed following several methodologies, essentially aimed at assessing the stability of the structure under static and seismic loads.

Algorithms able to define the collapse mechanism of arches and vaults were developed following the Heyman's theory and an approach based on rigid-brittle elements [1-3]. However, the definition of the collapse mechanism and the collapse multiplier can also be pursued with traditional finite element methods, according to nonlinear procedures that take into account the progressive damage of the structures. In case of seismic actions, this type of incremental analysis, proposed as an alternative to limit analysis in recent standards [4], is referred to as “static nonlinear analysis” or “pushover analysis”.

Among all the methodologies that can be used for the analysis of arches, modelling through spatial one-dimensional finite elements (frame elements) is particularly interesting. It allows to assess the arch static and seismic behaviour with low computational effort and ease of use. Frame elements allow for complete control over the analysis results and accurate vulnerability asassessments.

A previous work [5] introduced the “block-joint” model, where both blocks and joints are frame elements. The blocks have the mechanical properties of the stone and their cross section is the effective cross section of the arch, while the joints represent the mortar between the blocks and their cross section is ¼ of the one of the blocks. Each block interface is modelled through four joints placed at each vertex and connected to the block through rigid links. As shown in Figure 1, the joints are pin-fixed frame elements, the fixed-end provides continuity with the previous block, the pin-end transfers shear and axial force to the next block, in this manner the system transfers moment, shear and axial force from one block to another.

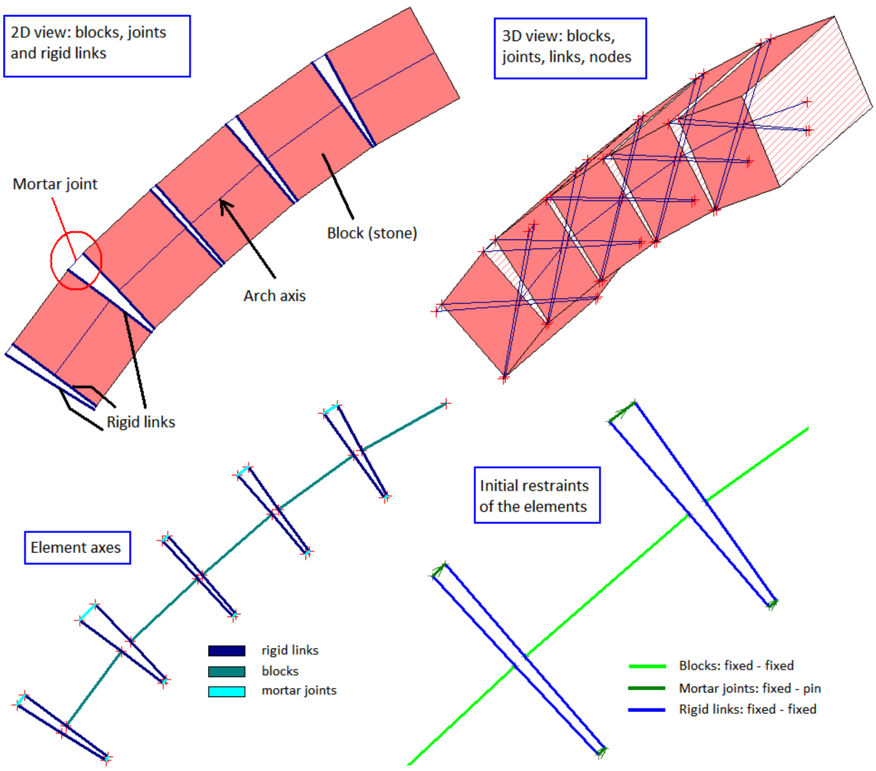

Figure 1. The finite element model block-joint

The “block-joint” model catches the arch in-plane and out-of-plane behaviour detecting the formation of plastic hinges where the tensile stresses overcome the resistance of the material.

Particularly interesting is the behaviour of the structure under horizontal inertial loads, especially when assessing monumental buildings in seismic prone areas. In these cases the structure is studied through static nonlinear analysis. An increasing horizontal force is applied to the system recording at each step the value of the control displacement, this leads to a force-displacement diagram that represents the capacity curve of the structure.

Performing a pushover analysis, increasing horizontal forces are applied to the system. At each step axial force verification is applied to the joints: if tensile stresses occur, the internal axial force is released and the fixed-end is turned to pin so that the element loses any stiffness and the internal actions remain constant at the value reached so far. As the analysis continues, the progressive deterioration of the joints leads to an unstable configuration that defines the end of the pushover curve.

Assuming that the internal actions remain constant after deterioration of the joints, this procedure is capable to easily find a balanced solution compatible with the mechanical characteristics of the materials. The procedure can be applied considering or not a limited tensile strength of the material: if the tensile strength is assumed to be null a comparison with the rigid-brittle limit analysis is feasible. However, accounting for the tensile strength of the joints surely leads to an increase of the collapse multiplier and allows not to underestimate the seismic capacity of the structure.

Besides the axial force verification, at each step of the pushover analysis compression and shear verifications can be applied to the blocks. As regards the compression verification, if the action overcomes the resistance of the material, the axial force is released. As regards the shear, if the axial force to shear ratio exceeds the friction coefficient of the material, the relevant frame turns into pin-pin and reacts only to axial force.

If the object of the analysis is a building that features arches and/or vaults, once the capacity curve has been calculated, the safety verification can be performed with the classic methodologies provided by the standards for pushover analysis.

Whereas if the object of the analysis is a macroelement, that is a portion of the building, the safety verification should follow the method provided by the standards for the kinematic analysis of collapse mechanisms [6]. In this case the collapse multiplier is pursued as the maximum static multiplier instead of the minimum kinematic multiplier.

Recent improvements of the method introduced: (a) the blocks modelled as curved beam elements, (b) a self-correcting procedure at the beginning of the pushover analysis in order to pursuit a stable configuration under static loads, (c) a solution for the automatic distribution of the vertical loads, (d) the possibility to account for passive and active strengthening techniques.

Moreover, the research for the development of the block-joint method led to the introduction of a simplified model that captures the behaviour of the arch with a lower number of finite elements: the “block-block” model. In fact, the arches modelled with the block-joint method may feature a high number of frame elements (blocks, joints, rigid links) depending on the discretization adopted, therefore this method is recommended for advanced analysis of macroelements, whereas the simpler block-block method can be used for the modelling of arches within complex buildings.

2 BLOCKS MODELLED AS SPATIAL CURVED BEAM

Modelling the blocks with curved elements, instead of straight frame elements, leads to certain advantages: (a) arches discretized in few blocks can be modelled adequately, (b) the mortar joints between the blocks can be short enough both at the intrados and at the extrados of the arch, (c) the self-weight of the blocks is better approximated.

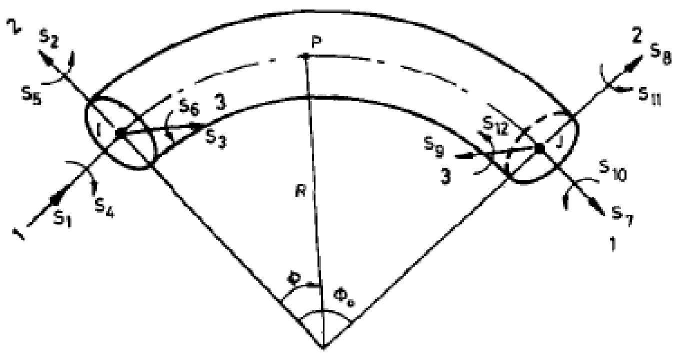

Figure 2. A curved beam element with coordinates and forces

As shown in Figure 2, formulation of the stiffness matrix of a spatial beam element with circular axis [7], validated through analytical formulas [8], was adopted for the blocks. The circular axis allows for a better modelling of several arch typologies such as semicircular arch, segmental arch, pointed arch, three-centered arch.

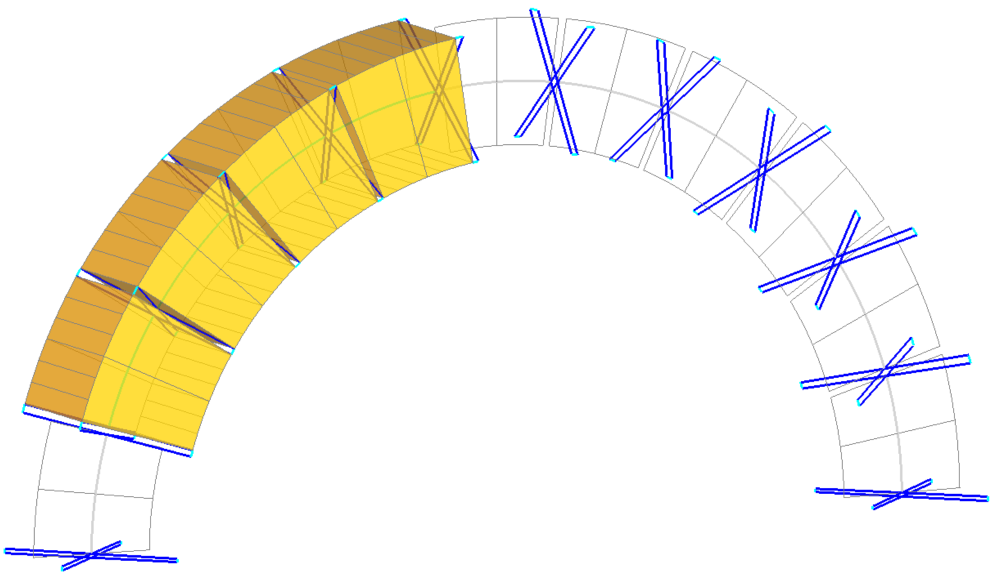

Figure 3. Block-joint modelling with curved beams

3 BLOCK-BLOCK MODELLING

Modelling of masonry buildings is frequently rather complex, a building that features many arches may need a large computational effort. Let us call "module" the portion of the arch between two consecutive mortar joints derived from arch discretization. The block-joint model consists of 14 frame elements per module (2 blocks + 4 joints + 8 rigid links), whereas the simplified block-block method allows modelling of the arch with only 2 blocks per module. Each module consists at least of two blocks because the self-weight is applied as point load in the central node of each module.

Figure 4 shows the case an arch discretized in 11 modules. The block-joint model of the arch consists of 200 frames and 155 nodes, while the block-block model of the same arch consists of 56 frames and 47 nodes. Therefore, the block-block model leads to a number of elements 3 times smaller than the block-joint model, with clear advantages with respect to the computational effort. For this reasons the block-block method is recommended for the modelling of arches within complex buildings.

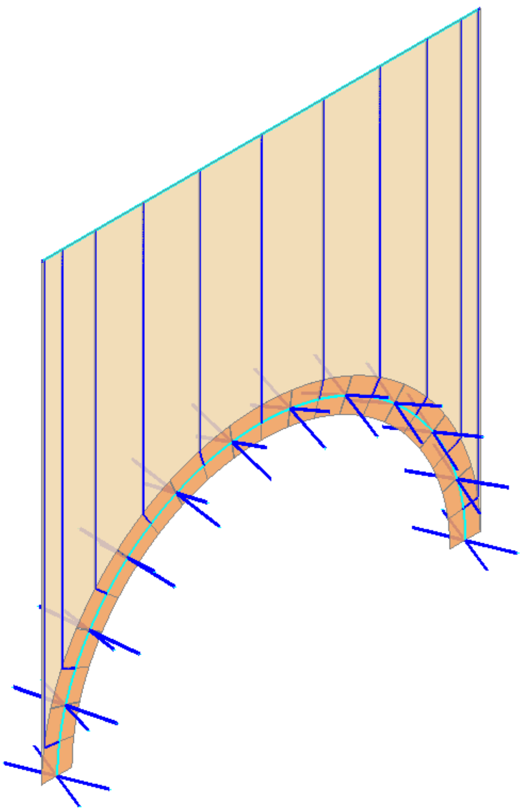

Figure 4. (a) block-joint model. (b) block-block model

The block-block method provides adequate verification procedures in order to achieve solutions compatible with the ones obtained with more advanced modelling. The blocks are spatial curved beam elements with fixed-fixed ends. The compressive strength of the blocks is the one of the stone while the tensile strength is the one of the mortar.

Performing pushover analysis, at each step a thrust-line verification is applied to the blocks. If the thrust-line comes out of the block cross section and the tensile stress exceeds the material tensile strength (and/or the reinforcement resistance) the verification is not satisfied, thus the bending moment is released by introducing a hinge in the relevant end of the block. In this way at the next steps of the analysis the position of the thrust-line remains practically the same: small variation may occur due to the variation of the axial force.

In a manner similar to the block-joint method, the compression and shear verification can be applied to complete the arch safety assessment.

4 SELF-CORRECTING GRAVITY ANALYSIS AT THE BEGINNING OF THE PUSHOVER ANALYSIS

Whatever the modelling method, the static asset, that is the resistant configuration under vertical static loads, is very important for the structural analysis of the arches. This asset matches the one at the beginning of the pushover analysis. It should be noted that the arch may develop hinges even if the only applied loads are the static ones. This simply means that the structure bears the static loads with a reduced degree of hiperstaticity. Therefore, a correct static nonlinear analysis must initially perform a self-correcting gravity analysis that pursuit a stable configuration of the arch under static loads .

The self-correcting procedure performs iteratively a static analysis and, at each iteration, it applies verification to the joints (or the blocks in the block-block model) and corrects the one in the most severe condition acting on its internal releases. In this way, at each step the arch loses one degree of hiperstaticity. The procedure continues until all the elements satisfy the verifications, then the pushover analysis can be performed on the resulting structure.

As regards the block-joint model, the corrections applied to the relevant joint during the gravity analysis consist in releasing the axial force and the bending moment at the two ends. Instead, as regards the block-block model, the bending moment is released in the section under the most sever conditions and two moments are applied at the adjacent blocks in order to fix the thrust-line in a position compatible with the geometry of the arch, the resistance of the material and eventually the presence of reinforcements.

During the iterative procedure and the application of several releases the structure may become unstable, that means that the structure cannot bear the static loads, thus the pushover analysis cannot be performed. For the examples described at section 7 of this work, the pushover analysis has been performed applying the self-correcting gravity analysis.

5 VERTICAL LOADS DISTRIBUTION

Frequently, arches are located within buildings and the modelling should account for the interaction with the adjacent structures such as masonry piers, columns, spandrels and beams. In the “Equivalent Frame” models the connections between different typologies can be set in order to achieve the correct distribution of the vertical loads.

Generally, the vertical loads acting on the structure above the arch are transferred to the arch through the spandrels. The solution proposed in this work (compatible with the equivalent frame model) consist of a series of rigid vertical struts that link arch and the beam above the arch providing the correct distribution of the vertical loads. The method has been validated through comparison between the loads directly applied to the blocks and the ones transferred by the struts through axial force [9, 10].

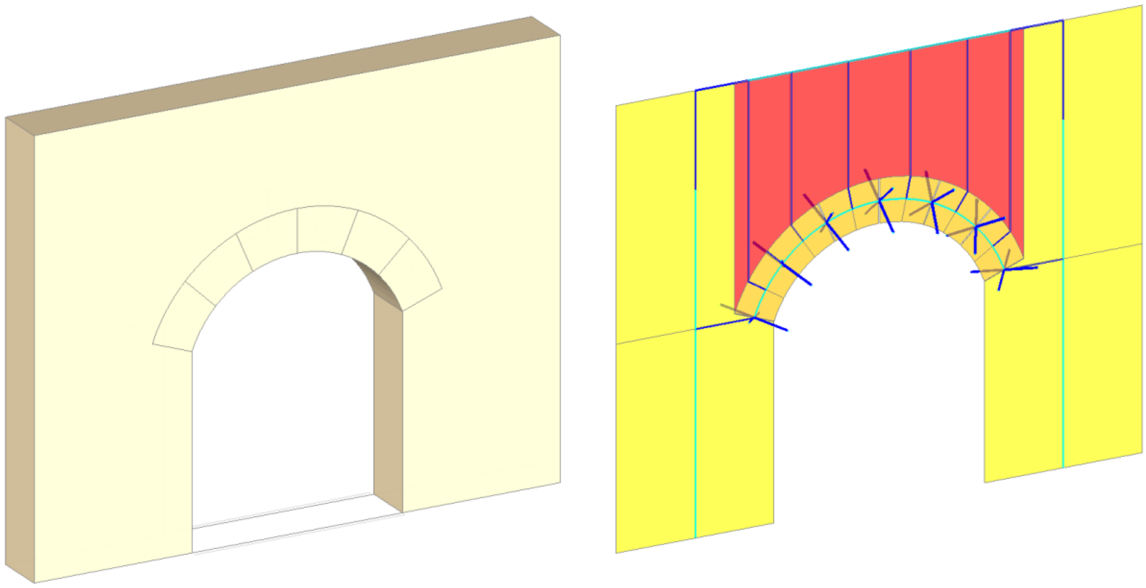

Figure 5. Rigid vertical struts for load distribution

More in details, the vertical struts connect the central node of each arch module to the above beam and feature a hinge at the interface with the arch, while the frame elements of the beam are pin-pin and carry a uniform load that represent the self-weight of the structure between consecutive struts. This system transfers to the arch the entire load of the above structure.

However, in real cases, a part of the load tends to migrate towards the adjacent masonry piers. In order to account for this behaviour the original configuration of the struts and the beam can be modified as follows: (a) exclude from the model some of the struts at the sides of the arch, (b) set as fixed-fixed the relevant frame elements of the beam.

Figure 6 shows the position of the nodal masses (red spheres) and highlights the portion of the structure that transfers load to the arch.

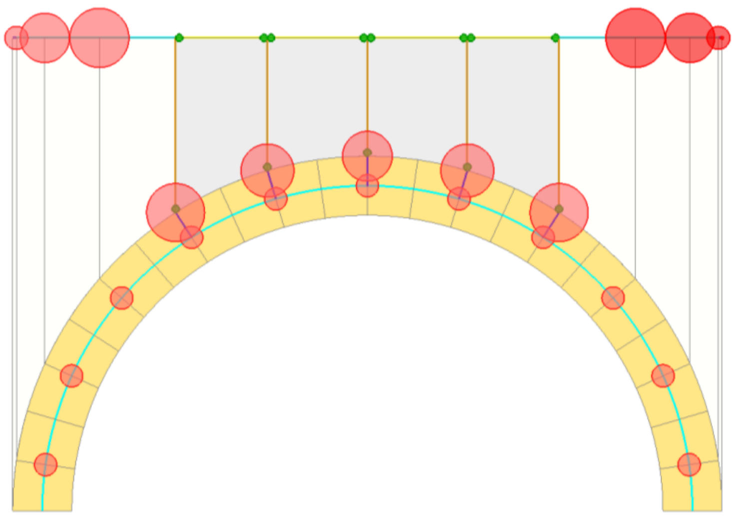

Figure 6. Partial load distribution

6 ACTIVE AND PASSIVE STRENGTHENING MEASURES

As mentioned before, the block-joint and the block-block method can account for a tensile strength of the mortar joints improving the stability of the arch. Moreover, besides the resistance of the material, the method can account for the benefic effects of passive and active strengthening measure.

The passive interventions such as FRP sheets applied at the intrados or at extrados of the arch provide an additional tensile resistance to blocks and joints, whereas the active interventions such as the “reinforced arch” technique, apply a prestress state in the arch aimed to bring the thrust-line closer to the central axis and to the formation of the plastic hinges [11].

An application of these strengthening typologies is provided in the next section.

7 APPLICATION EXAMPLES

The following examples, developed using Aedes Software PCM and SAV, apply the proposed methods [9, 10].

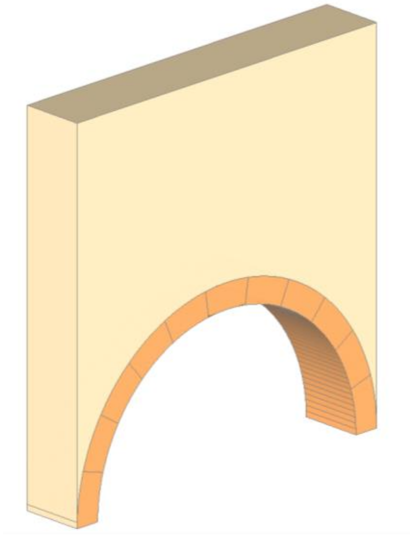

Figure 7. Reference arch

The reference arch shown in Figure 7 is semi-circular with the following characteristics: span 5.00 m, rise 2.50 m, depth 1.00 m, thickness 0.40 m. The height of the structure above the extrados of the arch is 3.10 m. The mechanical characteristics of the materials are shown in Table 1.

Masonry	Specific weight	19	kN/m^3
Blocks	Specific weight	20	kN/m^3
	Elastic modulus	50000	N/mm^2
	Shear modulus	20000	N/mm^2
	Compressive strength	35	N/mm^2
Joints	Elastic modulus	660	N/mm^2
	Shear modulus	264	N/mm^2
	Tensile strength	0.05	N/mm^2

Table 1. Mechanical properties of the materials

7.1 Vertical load distribution

This example aims to validate the distribution of the vertical load used in the block-joint and block-block methods through a comparison with the rigid-brittle method. Aedes SAV was used to model the arch according to the rigid-brittle method, the load arising from the structure above the arch is directly applied to the blocks as vertical point loads following the scheme shown in Figure 8a. The point loads applied to the blocks range from 15.98 kN at the imposts to 50.33 kN at the keystones.

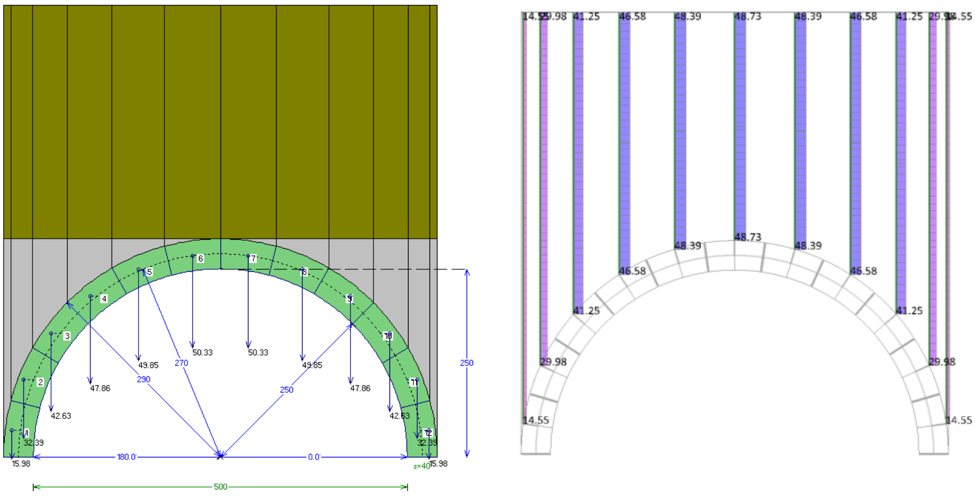

Figure 8. Vertical load distribution: (a) rigid-brittle model, (b) block-joint model

On the other hand, in the proposed methods (block-joint or block-block), the load is transferred to the arch through vertical rigid struts. Aedes PCM was used to model the arch according to the block-joint method. A static analysis was performed and, as shown in Figure 8b, the axial force in the vertical struts range from 14.55 kN at the imposts to 48.73 at the keystone. The values are very similar to the loads applied in the rigid-brittle method; therefore the use of vertical struts for load distribution is appropriate.

The rigid-brittle approach based on the Heyman's theory [2, 3] has been widely used for the structural analysis of arches [1], thus it represents a good reference for the calibration of alternative methodologies. Figure 9 shows the static analysis results (axial force diagram and thrust-line) obtained with the rigid-brittle approach and with the block-joint method.

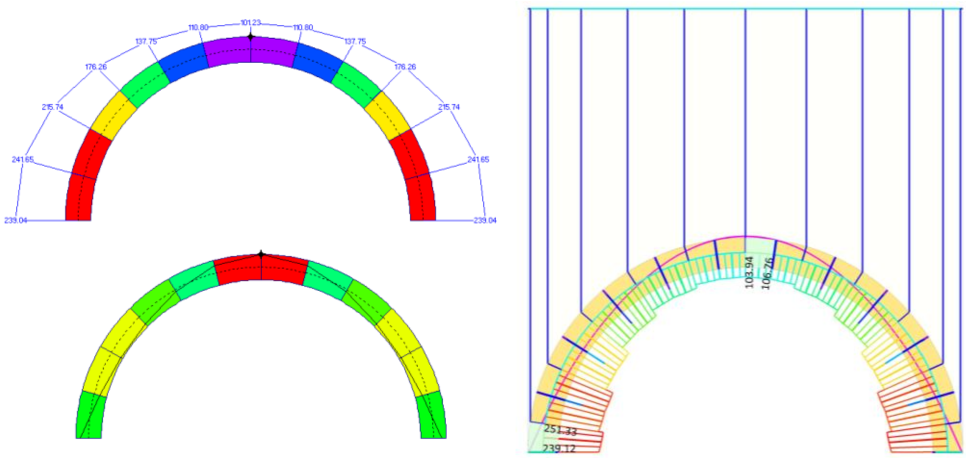

Figure 9. Static analysis: axial force diagram and thrust-line. (a) rigid-brittle approach. (b) block-joint method

Model	Vertical reaction	Horizontal reaction
Rigid-Brittle	239.04 kN	101.23 kN
Block-Joint	239.06 kN	103.92 kN

Table 2. External reactions at the imposts

Table 2 lists the vertical and horizontal external reactions at the fixed imposts. The analyses performed with the two approaches led to similar results and this validates the use of the block-joint method.

7.2 Comparison between block-joint and block-block methods

The reference arch is now modelled according to the two proposed methods (block-joint and block-block) performing modal, static and pushover FEM analyses. The following figures and tables show the results of the analyses performed.

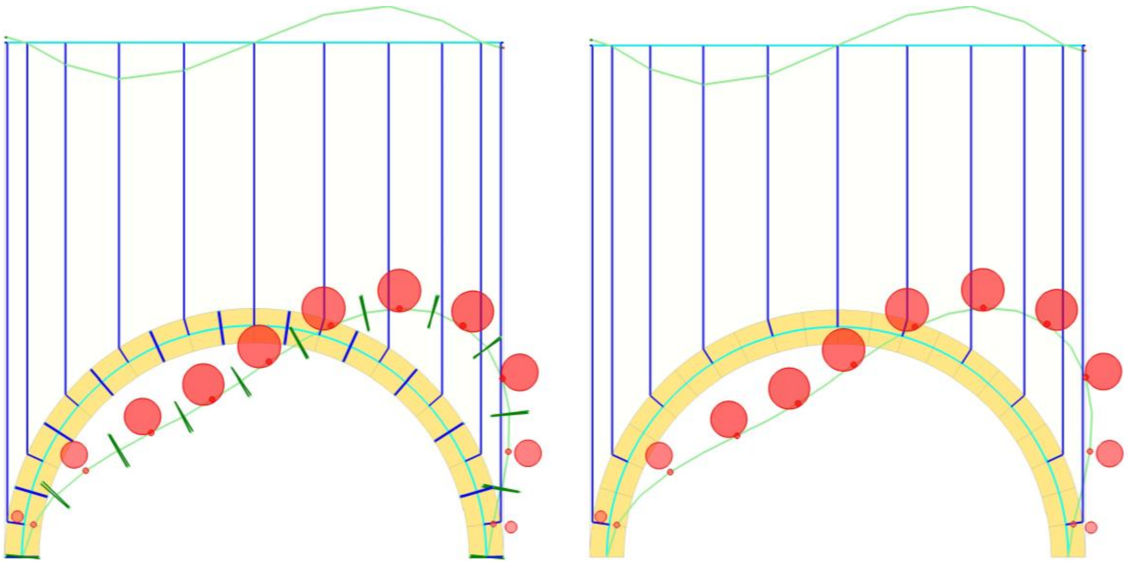

Figure 10. Modal analysis. (a) block-joint model, T = 0.060s. (b) block-block model, T = 0.050s

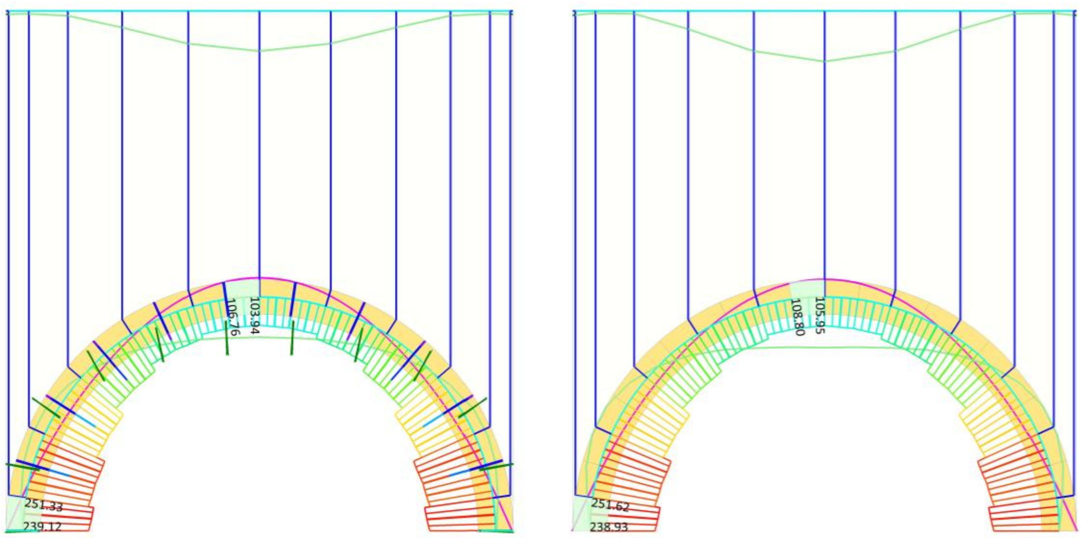

Figure 11. Static analysis. (a) block-joint model. (b) block-block model

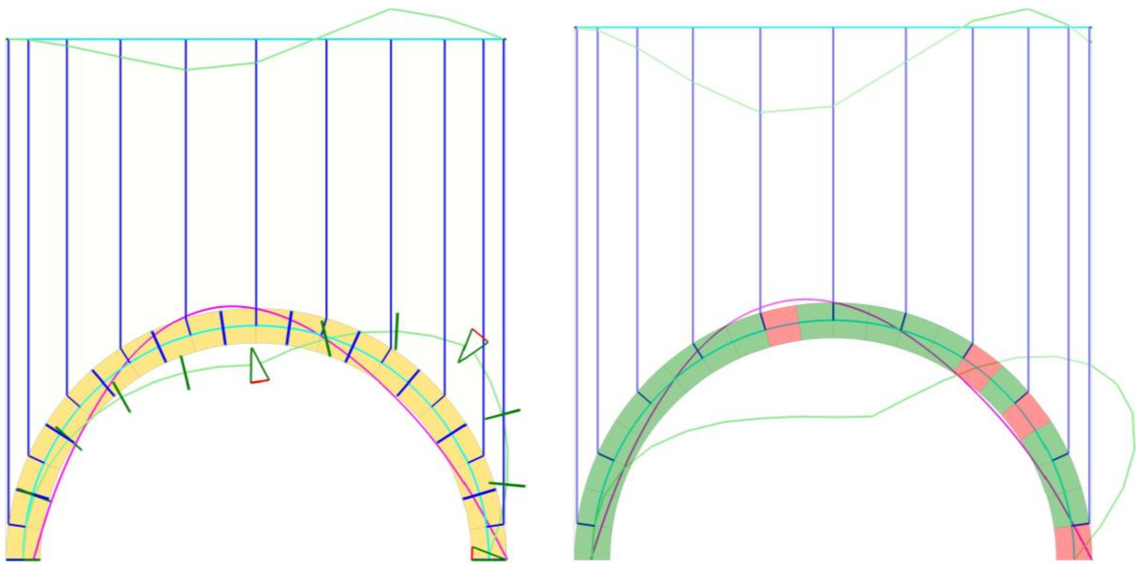

Figure 12. Pushover analysis. (a) block-joint model. (b) block-block model

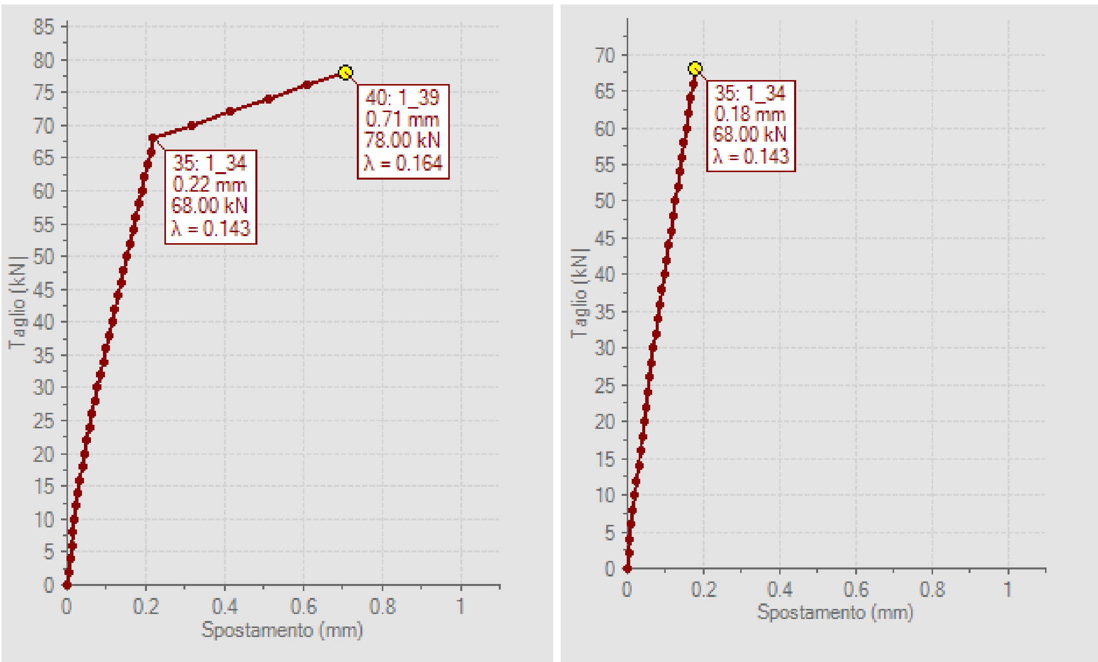

Figure 13. Pushover analysis, capacity curve. (a) block-joint model. (b) block-block model

The capacity curve obtained with the block-joint model features a plastic phase that lacks in the block-block model. The difference is certainly due to the higher accuracy of the block-joint with respect to the block-block modelling but it is not quite relevant for the seismic assessment of the arch, being the total force resistance and the position of the plastic hinges highly comparable.

7.3 Strengthening interventions

In this example passive and active strengthening interventions are applied to te reference arch and their efficiency is assessed through pushover analysis performed on the block-joint model.

As regards the passive interventions, we consider the application of FRP sheets at the intrados of the arch by increasing the tensile strength of the relevant joints according to the capacity of the intervention.

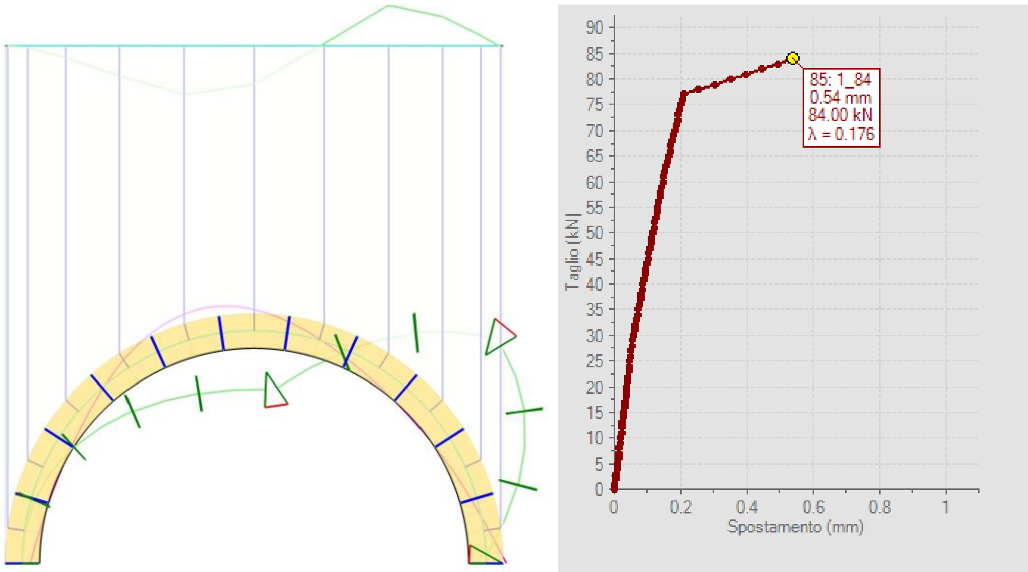

Figure 14. Pushover analysis, passive interventions (a) Thrust-line and displacement (b) capacity curve

As regards the active interventions, a prestress state is applied by means of ties placed along the intrados of the arch. The effect of the intervention is modelled with the application of radial point loads in the centroid of each block as shown in the following figure. This brings the thrust-lone closer to the central axis of the arch, thus it delays the formation of plastic hinges.

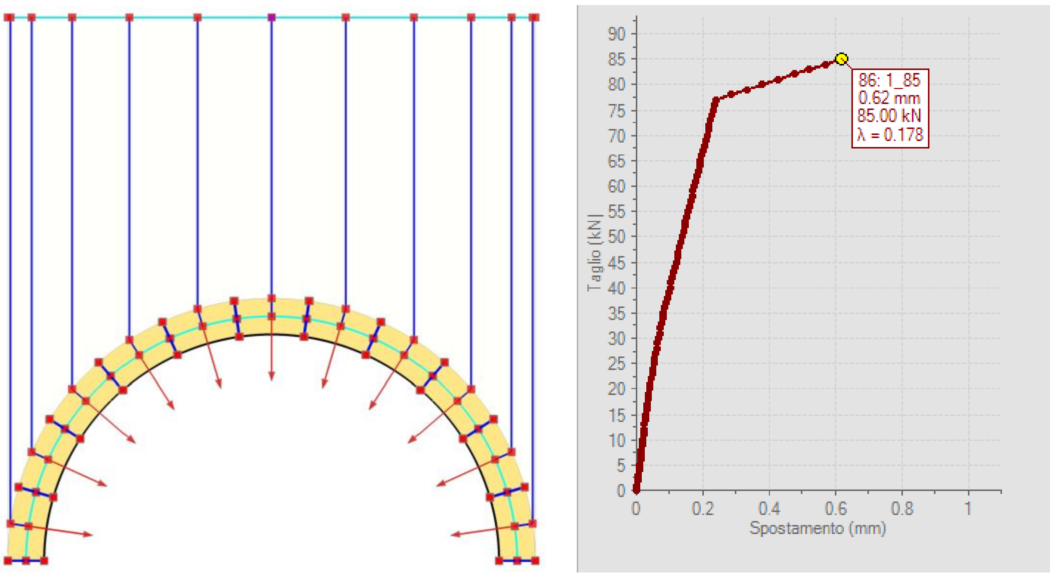

Figure 15. Active interventions (a) Radial point loads (b) Pushover analysis, capacity curve

Both the interventions considered led to a substantial improvement of the structural capacity of the arch.

8 CONCLUSIONS

This paper introduced the block-block model, a new method for the structural analysis of arches and vaults. The technique has been developed on the base of the block-joint method and aims to analyse arches located within macroelements or complex building through a simplified model. Both block-block and block-joint models allow for the correct assessment of the structural behaviour of the arch. The pushover analysis performed with the two methods leads to similar results.

Moreover the following aspects have been introduced and validated: (a) the blocks are now modelled with spatial curved beam elements, (b) at the beginning of the pushover analysis a self-correcting gravity analysis pursuits a stable configuration of the arch under static loads acting on the internal releases of the elements, (c) the vertical loads arising from the structure above the arch are transferred to the arch through vertical rigid struts, (d) the effect of passive and active strengthening interventions is accounted for in the methods proposed.

Several application examples have been presented to describe the proposed methodology.

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